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On the generation of Abelian magnetic fields in $SU(3)$ gluodynamics at high temperature

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Abstract. The vacuum state of $SU(3)$ gluodynamics at high temperature is investigated. A consistent approach including the calculation of the spontaneously generated constant chromomagnetic isotopic H_3 and hypercharge H_8 fields and the polarization operator of charged gluons in this background is applied. It is shown within the effective potential, taking into consideration the one-loop plus daisy diagrams, that the specific values of the fields yield a global minimum to the free energy. The spectrum of the transversal charged modes is stable at high temperature due to the calculated gluon magnetic mass which accounts for the fields. This leads to stable chromomagnetic fields in the deconfinement phase of QCD. A comparison with results of other approaches is made.

1 Introduction

The deconfinement phase transition remains the most topical problem of QCD at finite temperature. Nowadays a general belief is that the formation of the magnetic monopole condensate at low temperature and its evaporation at high temperature are responsible for this phenomenon (see, for instance, [1–5] and references therein). This scenario has been investigated both on a lattice [1–4] and in a continuum quantum field theory [5]. The present day status of this problem is characterized by the fact that the results obtained by the latter method are in agreement with that of the former one and they complement each other, although some discrepancies exist. The most important discrepancy concerns the properties of the high-temperature phase. As is well known from lattice calculations, due to asymptotic freedom at high temperature the deconfinement phase is to be a gas of free quarks and gluons: a quark–gluon plasma. No other macroscopic parameters except temperature are expected. However, in the continuum calculations in [6–9] the generation of a classical chromomagnetic field of order $gH \sim g^4T^2$ was observed. This spontaneously created field is a reflection of the infrared dynamics of the non-Abelian gauge fields at finite temperature. In [6,8] the creation of the field was considered. The field stabilization has been investigated for $SU(2)$ gluodynamics. Two possible mechanisms were considered, one due to the electrostatic potential (so-called the A_0 condensate) [7] and one due to the radiation corrections to the charged gluon spectrum [10]. According to the picture derived in these papers the vacuum at high temperature is to be a stable magnetized state. The noted discrepancy as well as some properties of the vacuum at high temperature have been discussed recently by Meisinger and Ogilvie [11]. To eliminate the classical field in the deconfinement phase these authors have introduced a gluon magnetic mass on heuristic grounds. Then they observed that to have a zero field at high temperature the value of the magnetic mass substituted into the one-loop effective potential (EP) must be of order $\sim g^2T$. However, the spontaneous creation of chromomagnetic fields is related with the infrared properties of non-Abelian gauge fields [8], which should be taken into consideration. This important aspect of the gauge field dynamics at high temperature needs more detailed investigations. Moreover, to find a consistent picture at high temperature the correlation corrections accounting for long distance effects should be calculated [12].

In the present paper the restored phase of QCD at high temperature is investigated within the approach consisting of two-stage calculations. First, the EP of the Abelian constant chromomagnetic fields – the isotopic one, H_3, and the hypercharge one, H_8 – taking into consideration the one-loop and the daisy diagrams, which include the gluon magnetic mass insertions, is computed, and the field configuration, which is spontaneously generated, is determined. This potential is real due to the daisies of the charged gluons, which cancel the imaginary part entering the one-loop part of the EP. As it occurred, a specific combination of both fields is formed. Second, the one-loop polarization operator (PO) of charged gluons in these external fields is calculated. It is averaged over the gluon tree-level states, in order to find the radiation corrections to the spectrum. In this way the Debye's and magnetic masses of gluons are de-

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rived. Then the vacuum magnetic field strengths are used to check whether or not the charged gluon spectrum (and therefore the magnetized vacuum) at finite temperature is stable. As is found, this is the case and the non-trivial vacuum is favorable at high temperature in a wide interval of temperature above the deconfinement transition temperature T_d. Hence we come to the conclusion that the scenario with the spontaneously magnetized vacuum results is a consistent picture. The higher loop corrections can be included perturbatively.

This paper is organized as follows. In Sect. 2 the charged sector of the $SU_c(3)$ gluodynamics is introduced. In Sect. 3 the generation of the external chromomagnetic fields is considered and the field strengths are derived. In Sect. 4 we calculate the PO in the external Abelian chromomagnetic fields at finite temperature and carry out the high-temperature expansion of the one-loop radiation corrections to the Landau levels. The last section is devoted to a discussion of the results obtained and to possible applications. In particular, a gauge invariance is discussed in detail.

2 The model

The Lagrangian of $SU(3)$ gluodynamics reads [13]

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + L_{\rm gf} + L_{\rm gh}, \quad (1)$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^a_\mu A^a_\nu$ is the field strength tensor, A^a_μ is the potential of the gluon field, the f^{abc} are the $SU_c(3)$ structure constants, $a = 1,\ldots,8$. The metric is chosen to be Euclidian in order to consider the theory at $T \neq 0$ in the imaginary time formalism. The external chromomagnetic field is introduced by dividing the gluon field A^a_μ into the sum of the classic background field B^a_μ and the quantum field Q^a_μ,

$$A^a_\mu = B^a_\mu + Q^a_\mu. \quad (2)$$

We choose the external potential in the form $B^a_\mu = \delta^{a3}B_{3\mu} + \delta^{a8}B_{8\mu}$, where $B_{3\mu} = H_3\delta_{\mu 2}x_1$ and $B_{8\mu} = H_8\delta_{\mu 2}x_1$ correspond to constant chromomagnetic fields directed along the third axis in the Euclidean space and $a = 3$ and $a = 8$ in the color $SU_c(3)$-space, respectively: $F^{a\,\rm ext}_{\mu\nu} = \delta^{a3}F_{3\mu\nu} + \delta^{a8}F_{8\mu\nu}$, $F_{c12} = -F_{c21} = H_{\rm c}$, $c = 3,8$. The gauge fixing term in (1) is

$$L_{\rm gf} = -\frac{1}{2}(\partial_\mu Q^a_\mu + g f^{abc} B^b_\mu Q^c_\mu)^2, \quad (3)$$

and $L_{\rm gh}$ represents the ghost Lagrangian. The components Q^a_μ with $a = 1,2,4,5,6,7$ correspond to the charged gluons. It is convenient to introduce the "charged basis" of the fields Q^a_μ $(a = 1,2,4,5,6,7)$ by the expressions

$$W^\pm_{1\mu} = \frac{1}{\sqrt{2}}(Q^1_\mu \pm {\rm i}Q^2_\mu),\ W^\pm_{2\mu} = \frac{1}{\sqrt{2}}(Q^4_\mu \pm {\rm i}Q^5_\mu),$$

$$W^\pm_{3\mu} = \frac{1}{\sqrt{2}}(Q^6_\mu \pm {\rm i}Q^7_\mu). \quad (4)$$

After simple algebra one obtains the Lagrangian of the charged gluons in the form

$$\begin{aligned}
&L_{\rm ch.gl.}\\
&= \sum_{r=1}^{3}\Big(-\frac{1}{2}W^+_{r\mu\nu}W^-_{r\mu\nu} - (D^*_\mu W^+_{r\mu})(D_\nu W^-_{r\nu})\\
&\quad -\frac{1}{2}c_r g^2 W^+_{r\mu}W^-_{r\nu}W^+_{r\lambda}W^-_{r\rho}\Gamma_{\mu\nu\lambda\rho}\Big)\\
&+ {\rm i}g(F_{3\mu\nu} + Q^3_{\mu\nu})W^+_{1\mu}W^-_{1\nu} + {\rm i}gQ^3_\mu(W^+_{1\nu}(\partial_\mu W^-_{1\nu} - \partial_\nu W^-_{1\mu})\\
&\quad -({\rm h.c.}))\\
&+ {\rm i}\sqrt{\frac{3}{2}}g\left(\lambda_+ F_{8\mu\nu} + Q^8_{\mu\nu} + \frac{1}{\sqrt{6}}Q^3_{\mu\nu}\right)W^+_{2\mu}W^-_{2\nu}\\
&+ {\rm i}\sqrt{\frac{3}{2}}g\left(Q^8_\mu + \frac{1}{\sqrt{6}}Q^3_\mu\right)(W^+_{2\nu}(\partial_\mu W^-_{2\nu} - \partial_\nu W^-_{2\mu})\\
&\quad -({\rm h.c.}))\\
&+ {\rm i}\sqrt{\frac{3}{2}}g\left(\lambda_- F_{8\mu\nu} + Q^8_{\mu\nu} - \frac{1}{\sqrt{6}}Q^3_{\mu\nu}\right)W^+_{3\mu}W^-_{3\nu}\\
&+ {\rm i}\sqrt{\frac{3}{2}}g\left(Q^8_\mu - \frac{1}{\sqrt{6}}Q^3_\mu\right)(W^+_{3\nu}(\partial_\mu W^-_{3\nu} - \partial_\nu W^-_{3\mu})\\
&\quad -({\rm h.c.})) + L_{\rm gh},
\end{aligned} \quad (5)$$

where $W^+_{r\mu\nu} = D^*_{r\mu}W^+_{r\nu} - D^*_{r\nu}W^+_{r\mu}$, $W^-_{r\mu\nu} = D_{r\mu}W^-_{r\nu} - D_{r\nu}W^-_{r\mu}$; $D_{r=1\,\mu} = \partial_\mu + {\rm i}gB_{3\mu}$, $D_{r=2,3\,\mu} = \partial_\mu + {\rm i}\sqrt{\frac{3}{2}}\lambda_\pm gB_{8\mu}$ are covariant derivatives, $\Gamma_{\mu\nu\lambda\rho} = \delta_{\mu\nu}\delta_{\lambda\rho} - \delta_{\mu\lambda}\delta_{\nu\rho}$,

$$\lambda_\pm = 1 \pm \frac{1}{\sqrt{6}}\frac{H_3}{H_8};$$

$c_r = 1, \frac{7}{4}, \frac{5}{4}$ for $r = 1,2,3$, respectively. The Lagrangian (5) is the starting point of our analysis.

3 The spontaneous generation of chromomagnetic fields

First, let us investigate the spontaneous vacuum magnetization in high-temperature $SU_c(3)$ gluodynamics. The charged sector is described by the Lagrangian (5). For this purpose we apply the effective Lagrangian method.

The effective Lagrangian of constant chromomagnetic fields H_3 and H_8 at finite temperature can be written in the form

$$L_{\rm eff} = L^{(1)} + L^{(\rm ring)} + \ldots, \quad (6)$$

where the first term represents the one-loop contribution of charged gluons:

$$L^{(1)} = -\frac{gH_3}{2\pi\beta}\sum_{l=-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{{\rm d}p_3}{2\pi}\sum_{n,\sigma}\ln[\beta^2 G^{-1}_{r=1}(p_3, H_3, T)]$$

$$
\begin{aligned}
&- \sqrt{\frac{3}{2}}\lambda_+ \frac{gH_8}{2\pi\beta} \\
&\quad \times \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p_3}{2\pi} \sum_{n,\sigma} \ln[\beta^2 G^{-1}_{r=2}(p_3, H_3, H_8, T)] \\
&- \sqrt{\frac{3}{2}}\lambda_- \frac{gH_8}{2\pi\beta} \\
&\quad \times \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p_3}{2\pi} \sum_{n,\sigma} \ln[\beta^2 G^{-1}_{r=3}(p_3, H_3, H_8, T)].
\end{aligned} \tag{7}
$$

Here r marks the index of the charged basis (4), G_r is the corresponding propagator in the external fields H_3 and H_8. The second term in (6) presents the contribution of daisy or ring diagrams of the charged gluons,

$$
\begin{aligned}
L^{(\mathrm{ring})}_{\mathrm{ch}} &= -\frac{gH_3}{2\pi\beta} \\
&\times \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p_3}{2\pi} \sum_{n,\sigma} \ln[1 \\
&\quad + G_{r=1}(\epsilon_n^2, H_3, T) \Pi^{r=1}(H_3, T, n, \sigma)] \\
&- \sqrt{\frac{3}{2}}\lambda_+ \frac{gH_8}{2\pi\beta} \\
&\times \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p_3}{2\pi} \sum_{n,\sigma} \ln[1 \\
&\quad + G_{r=2}(\epsilon_n^2, H_3, H_8, T) \Pi^{r=2}(H_3, H_8, T, n, \sigma)] \\
&- \sqrt{\frac{3}{2}}\lambda_- \frac{gH_8}{2\pi\beta} \\
&\times \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p_3}{2\pi} \sum_{n,\sigma} \ln[1 \\
&\quad + G_{r=3}(\epsilon_n^2, H_3, H_8, T) \Pi^{r=3}(H_3, H_8, T, n, \sigma)],
\end{aligned} \tag{8}
$$

and of the neutral gluons,

$$
\begin{aligned}
L^{(\mathrm{ring})}_{\mathrm{neut}} &= -\frac{1}{2\beta} \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p_3}{2\pi} \ln[\omega_l^2 + \overline{p}^2 + \Pi'(H_3, H_8, T)] \\
&\quad - \frac{1}{2\beta} \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p_3}{2\pi} \ln[\omega_l^2 + \overline{p}^2 + \Pi''(H_3, H_8, T)].
\end{aligned} \tag{9}
$$

These expressions include the PO of charged gluons averaged over physical states, which are also dependent on $H = H_3, H_8$, the level number $n = 0, 1, \ldots$, the spin projection $\sigma = \pm 1$, and the Debye masses of the neutral gluons Q^3_μ, Q^8_μ,

$$\Pi'(H, T) = \Pi'_{00}(k = 0, H, T),$$

$$\Pi''(H, T) = \Pi''_{00}(k = 0, H, T),$$

respectively. The averaged values of charged gluon PO taken in the state $n = 0$ and $\sigma = +1$ give the magnetic masses of the transversal modes. The gauge dependence of this mass needs some discussion. In fact, the PO is a gauge dependent object. Its pole positions are gauge fixing independent. This fact follows from the Nielsen identities derived at finite temperature and zero field in [16]. We believe that the field presence does not change this property. The tree-level spectrum of charged gluons in the fields is also gauge fixing independent. Therefore we also believe, although it is difficult to check explicitly, that the average value of the PO in the ground state of this spectrum is gauge fixing independent. In the used calculation procedure – a simple daisy resummation – the described magnetic mass is used as a given parameter in the effective Lagrangian for temperature and magnetic field. In the minimum it is also gauge fixing independent. Hence, we believe that our calculations are gauge fixing independent. Of cause, all the results are obviously dependent on the choice of the directions $a = 3$ and $a = 8$ in the internal space. We shall discuss in the last section the influence of gauge invariance on the ground state properties following mainly to ideas of Feynman [17]. The quantities $\Pi'(H, T)$ and $\Pi''(H, T)$ are the zero-zero components of the corresponding neutral gluon polarization operators calculated in the external fields $H = H_3, H_8$ at finite temperature and taken at zero momentum. The ring contribution to the L_{eff} has to be calculated when the vacuum magnetization at non-zero temperature is investigated. These diagrams account for long range correlations at finite temperature [9].

The detailed evaluations of the one-loop effective Lagrangian in finite-temperature $SU(2)$ gluodynamics have been carried out in [8, 9]. Performing them in our case we arrive at the following result for the high-temperature limit of $L^{(1)}$:

$$
\begin{aligned}
L^{(1)} &= -\frac{H_3^2}{2} - \frac{11}{32}\frac{g^2}{\pi^2} H_3^2 \ln\left[\frac{T}{\mu}\right] + \frac{1}{3\pi}(gH_3)^{\frac{3}{2}} \frac{T}{3\pi} \\
&\quad - \frac{H_8^2}{2} - \frac{11}{16}\frac{g^2}{\pi^2} H_8^2 \ln\left[\frac{T}{\mu}\right] \\
&\quad + \left(\lambda_+^{\frac{3}{2}} + |\lambda_-|^{\frac{3}{2}}\right) \left(\frac{3}{2}\right)^{\frac{3}{4}} (gH_8)^{\frac{3}{2}} \frac{T}{3\pi} \\
&\quad + \mathrm{i}\left[(gH_3)^{\frac{3}{2}} + \left(\lambda_+^{\frac{3}{2}} + |\lambda_-|^{\frac{3}{2}}\right)\left(\frac{3}{2}\right)^{\frac{3}{4}} (gH_8)^{\frac{3}{2}}\right] \frac{T}{2\pi},
\end{aligned} \tag{10}
$$

where $T \gg \sqrt{gH_{3,\,8}} \gg \mu$; μ is the renormalization point. The imaginary part in the expression (10) signals a vacuum instability and must be considered carefully. Namely, as will be shown below, the inclusion of ring diagrams, L^{ring}, leads to canceling of the imaginary parts so that the whole expression L_{eff} becomes real. To see this, let us consider the contribution of the ring diagrams which correspond to the unstable modes of the charged gluons:

$$L^{(\mathrm{ring})}_{\mathrm{unstable}} = -\frac{gH_3}{2\pi\beta}$$

$$\times \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p_3}{2\pi} \ln[1 + (\omega_l^2 + p_3^2 - gH_3)^{-1} \Pi^{r=1}(H_3, T)]$$
$$- \sqrt{\frac{3}{2}}\lambda_+ \frac{gH_8}{2\pi\beta}$$
$$\times \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p_3}{2\pi} \ln\left[1 + \left(\omega_l^2 + p_3^2 - \sqrt{\frac{3}{2}}\lambda_+ gH_8\right)^{-1} \Pi^{r=2}(H_3, H_8, T)\right]$$
$$- \sqrt{\frac{3}{2}}\lambda_- \frac{gH_8}{2\pi\beta}$$
$$\times \sum_{l=-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\mathrm{d}p_3}{2\pi} \ln\left[1 + \left(\omega_l^2 + p_3^2 - \sqrt{\frac{3}{2}}\lambda_- gH_8\right)^{-1} \Pi^{r=3}(H_3, H_8, T)\right], \quad (11)$$

where $\omega_l = 2\pi l T$, $l = 0, \pm 1, \ldots$, are the Matsubara frequencies. To obtain (11) one has merely to put $n = 0$ and $\sigma = +1$ in the expression for $L_{\mathrm{ch}}^{\mathrm{ring}}$. An elementary integration gives

$$L_{\mathrm{unstable}}^{(\mathrm{ring})} = -\frac{gH_3T}{2\pi}[\Pi^{r=1}(H_3, T) - gH_3]^{\frac{1}{2}}$$
$$- \sqrt{\frac{3}{2}}\lambda_+ \frac{gH_8T}{2\pi}\left[\Pi^{r=2}(H_3, H_8, T) - \sqrt{\frac{3}{2}}\lambda_+ gH_8\right]^{\frac{1}{2}}$$
$$- \sqrt{\frac{3}{2}}\lambda_- \frac{gH_8T}{2\pi}\left[\Pi^{r=3}(H_3, H_8, T) - \sqrt{\frac{3}{2}}\lambda_- gH_8\right]^{\frac{1}{2}}$$
$$- \mathrm{i}\left[(gH_3)^{\frac{3}{2}} + \left(\lambda_+^{\frac{3}{2}} + |\lambda_-|^{\frac{3}{2}}\right)\left(\frac{3}{2}\right)^{\frac{3}{4}}(gH_8)^{\frac{3}{2}}\right]\frac{T}{2\pi}. \quad (12)$$

From (10) and (12) it is seen that the imaginary parts are cancelled out in the total. The final effective Lagrangian L_{eff} is real if the relations

$$\Pi^{r=1}(H_3, T) > gH_3,$$
$$\Pi^{r=2}(H_3, H_8, T) > \sqrt{\frac{3}{2}}\lambda_+ gH_8$$

and

$$\Pi^{r=3}(H_3, H_8, T) > \sqrt{\frac{3}{2}}\lambda_- gH_8$$

hold.

In one-loop order the neutral gluon contribution is a trivial H-independent constant which can be omitted. However, these fields are long range states and give an H-dependent effective Lagrangian through the correlation corrections (9) depending on the temperature and external fields. Below, only the longitudinal neutral modes are included because their Debye's masses are non-zero. The corresponding effective Lagrangian is easily calculated and has the form [9]

$$L_{\mathrm{neut}}^{(\mathrm{ring})} = -\frac{T^2}{24}[\Pi'(H_3, H_8, T) + \Pi''(H_3, H_8, T)]$$
$$+ \frac{T}{12\pi}[\Pi'(H_3, H_8, T) + \Pi''(H_3, H_8, T)]^{\frac{3}{2}}$$
$$- \frac{1}{32\pi^2}[\Pi'(H_3, H_8, T) + \Pi''(H_3, H_8, T)]^2$$
$$\times \left(\log\left[4\pi T[\Pi'(H_3, H_8, T) + \Pi''(H_3, H_8, T)]^{-\frac{1}{2}} + \frac{3}{4} - \gamma\right]\right). \quad (13)$$

Evaluating the Debye masses of the neutral gluons Q_μ^3, Q_μ^8 gives the following results (see for details [9]):

$$(m_D')^2 = \Pi_{00}'(k = 0, H, T)$$
$$= \frac{8}{3}g^2T^2 - \frac{g^2T}{\pi} \times \left[(gH_3)^{\frac{1}{2}} + \left(\lambda_+^{\frac{1}{2}} + |\lambda_-|^{\frac{1}{2}}\right)\left(\frac{3}{2}\right)^{\frac{1}{4}}(gH_8)^{\frac{1}{2}}\right]$$
$$(m_D'')^2 = \Pi_{00}''(k = 0, H, T) \quad (14)$$
$$= 2g^2T^2 - \frac{g^2T}{\pi}\left(\lambda_+^{\frac{1}{2}} + |\lambda_-|^{\frac{1}{2}}\right)\left(\frac{3}{2}\right)^{\frac{1}{4}}(gH_8)^{\frac{1}{2}}.$$

Substituting the expressions (14) into $L_{\mathrm{neut}}^{(\mathrm{ring})}$, we obtain the correlation corrections due to the neutral gluons:

$$L_{\mathrm{neut}}^{(\mathrm{ring})} = \frac{g^2T^3}{24\pi}\left[(gH_3)^{\frac{1}{2}} + 2\left(\lambda_+^{\frac{1}{2}} + |\lambda_-|^{\frac{1}{2}}\right)\left(\frac{3}{2}\right)^{\frac{1}{4}}(gH_8)^{\frac{1}{2}}\right], \quad (15)$$

where the H-independent terms were skipped. Thus, the vacuum magnetization at high temperature $T \gg \sqrt{gH_{3,8}}$ will be investigated within the following effective Lagrangian:

$$L^{(\mathrm{eff})} = -\frac{H_3}{2} - \frac{H_8}{2} - \frac{11}{32}\frac{g^2}{\pi^2}H_3^2\ln\left[\frac{T}{\mu}\right] - \frac{11}{16}\frac{g^2}{\pi^2}H_8^2\ln\left[\frac{T}{\mu}\right]$$
$$+ \left[(gH_3)^{\frac{3}{2}} + \left(\lambda_+^{\frac{3}{2}} + |\lambda_-|^{\frac{3}{2}}\right)\left(\frac{3}{2}\right)^{\frac{3}{4}}(gH_8)^{\frac{3}{2}}\right]\frac{T}{3\pi}$$
$$- \frac{gH_3T}{2\pi}[\Pi^{r=1}(H_3, T) - gH_3]^{\frac{1}{2}}$$
$$- \sqrt{\frac{3}{2}}\lambda_+ \frac{gH_8T}{2\pi}\left[\Pi^{r=2}(H_3, H_8, T) - \sqrt{\frac{3}{2}}\lambda_+ gH_8\right]^{\frac{1}{2}}$$

$$- \sqrt{\frac{3}{2}}\lambda_- \frac{gH_8 T}{2\pi}\left[\Pi^{r=3}(H_3,H_8,T) - \sqrt{\frac{3}{2}}\lambda_- gH_8\right]^{\frac{1}{2}}$$
$$+ \frac{g^2T^3}{24\pi}\left[(gH_3)^{\frac{1}{2}} + 2\left(\lambda_+^{\frac{1}{2}} + |\lambda_-|^{\frac{1}{2}}\right)\left(\frac{3}{2}\right)^{\frac{1}{4}}(gH_8)^{\frac{1}{2}}\right]$$
$$+ O(g^3). \quad (16)$$

This expression includes the contributions of $L^{(1)}$ as well as $L^{\text{ring}}_{\text{unstable}}$ and $L^{\text{ring}}_{\text{neut}}$. Notice that the quantity $L^{\text{ring}}_{\text{unstable}}$ has the order $g^{\frac{9}{4}}$ in coupling constant g, whereas the order of $L^{\text{ring}}_{\text{neut}}$ is $g^{\frac{5}{2}}$. In other words, the contribution of the neutral gluons does not play an essential role in the generation of external fields and this part can be dropped. This is natural on general grounds because the neutral gluon field is stable at the tree and the one-loop levels. So, one does not have to expect any role of this sector in the field generation. Thus, in this approximation, $L^{(\text{eff})}$ is equal to the effective Lagrangian (16) without the terms in the last line.

Our problem is divided into two separate parts: first, one has to calculate the spontaneously generated fields in the vacuum and, second, to compute $\Pi^r(H,T,n,\sigma)$, which are the average values of the charged gluon PO taken in the tree-level states.

To derive the strengths of the generated fields one has to solve the set of stationary equations

$$\frac{\partial L^{(\text{eff})}}{\partial H_3} = 0,$$
$$\frac{\partial L^{(\text{eff})}}{\partial H_8} = 0.$$

There are three non-trivial solutions:

$$H_3 = 0, \quad H_8 = \left(\frac{3}{2}\right)^{\frac{3}{2}} \frac{g^3T^2}{\pi^2}, \quad (17)$$

$$H_3 = \frac{1}{4}\left(1+\frac{1}{\sqrt{2}}\right)^2 \frac{g^3T^2}{\pi^2}, \quad H_8 = 0 \quad (18)$$

and

$$H_3 = 0.2976\,\frac{g^3T^2}{\pi^2},$$
$$H_8 = 0.9989\left(\frac{3}{2}\right)^{\frac{3}{2}}\frac{g^3T^2}{\pi^2}. \quad (19)$$

The terms of L^{eff} that depend on the magnetic masses of the transversal modes are not included in the expressions (17)–(19), because their contributions are of higher order in g. Note that the latter of these configurations corresponds to the minimum of the EP and we, therefore, conclude that both chromomagnetic fields have to arise spontaneously at high temperature.

In determining the solutions (17)– (19), the logarithmic terms $\sim \ln\left[\frac{T}{\mu}\right]$ were omitted as negligibly small. This approximation is appropriate for the region $T \geq T_{\text{d}}$, where T_{d} is the deconfinement phase transition temperature. However, in the limit $T \to \infty$, the logarithmic terms become large and should be accounted for. To analyze the asymptotic region the solutions of the above stationary equations must be rewritten in terms of the effective coupling constant:

$$g^2_{\text{eff}} \approx \left(\frac{11}{16\pi^2}\ln\left[\frac{T}{\mu}\right]\right)^{-1}.$$

For this purpose one has merely to eliminate the tree-level terms in the $L^{(\text{eff})}$. It turns out that, at $T \gg T_{\text{d}}$, the field configuration

$$gH_3 = \frac{1}{4}\left(1+\frac{1}{\sqrt{2}}\right)^2 \frac{g^4_{\text{eff}}T^2}{\pi^2}, \quad gH_8 = 0 \quad (20)$$

yields the global minimum of the EP. That is, only the isotopic chromomagnetic field is generated at asymptotically high temperatures. If the temperature decreases, the hypercharge H_8 field appears below some temperature T_0, and in the deconfinement region $T_0 > T \geq T_{\text{d}}$, where the terms $\sim \ln\left[\frac{T}{\mu}\right]$ are small in comparison with the tree-level ones, both fields are present.

4 Gluon polarization operator

The next question that must be answered is whether the chromomagnetic fields H_3 and H_8 obtained in (19) are stable. This complicated problem requires the explicit calculation of the charged gluon polarization operator in the external fields H_3 and H_8.

In one-loop order the PO is determined by the standard set of diagrams in Fig. 1, where double wavy lines represent the Green function $G_{r\,\mu\nu}(x,y)$ for the charged gluons, dashed double lines correspond to the Green function $D(x,y)$ for the charged ghost fields. Thin wavy and thin dashed lines stand for the neutral gluon fields $Q^{3,8}_\mu$ and the neutral ghost fields $C_{3,8}$, respectively. In the operator form the above Green functions are given by the expressions (in Feynman's gauge)

$$G_{r=1\,\mu\nu}(P) = -[P^2 + 2\mathrm{i}gF_{3\mu\nu}]^{-1},$$

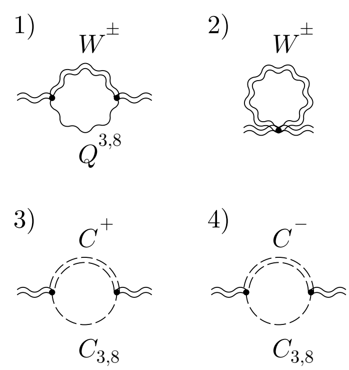

Fig. 1. Polarization operator of charged gluons in the one-loop approximation

$$G_{r=2,3\,\mu\nu}(P) = -[P^2 + \sqrt{6}\mathrm{i}\lambda_\pm g F_{8\mu\nu}]^{-1},$$
$$D(P) = -\frac{1}{P^2}.$$

To calculate the PO we make use of the proper time representation and the Schwinger operator formalism [14]. The PO of charged gluons in a chromomagnetic field at non-zero temperature can be written as

$$\Pi^r_{\mu\nu} = \frac{g^2}{\beta}\, c_r \sum_{k_4} \int \frac{\mathrm{d}^3k}{(2\pi)^3} \Pi^r_{\mu\nu}(k,P), \quad r = 1,2,3, \quad (21)$$

where

$$\begin{aligned}
&\Pi^r_{\mu\nu}(k,P) \\
&= k^{-2}\Big\{\Gamma_{\mu\alpha,\rho} G_{r\ \alpha\beta}(P-k)\Gamma_{\nu\beta,\rho} + (P-k)_\mu D(P-k)k_\nu \\
&+ k_\mu D(P-k)(P-k)_\nu \\
&+ k^2\Big[G_{r\ \mu\nu}(P-k) - 2G_{r\ \nu\mu}(P-k) \\
&\qquad + \delta_{\mu\nu} G_{r\ \rho\rho}(P-k)\Big]\Big\},
\end{aligned}$$
$$\Gamma_{\mu\alpha,\rho} = \delta_{\mu\alpha}(2P-k)_\rho + \delta_{\alpha\rho}(2k-P)_\mu + \delta_{\mu\rho}(P+k)_\alpha;$$

$\beta = \frac{1}{T}$, $k_4 = \frac{2\pi l}{\beta}$, $l = 0, \pm1, \pm2, \ldots$, $P_\mu = \mathrm{i}\partial_\mu + gB_{3\mu}$ for $r = 1$ and $P_\mu = \mathrm{i}\partial_\mu + \sqrt{\frac{3}{2}}\lambda_\pm g B_{8\mu}$ for $r = 2,3$, respectively, the constant c_r is defined above. We restrict our consideration to the case of the high-temperature limit. In (21) this limit corresponds to the $l = 0$ term in the sum over k_4 [12]. To evaluate the expression for the PO we used the Schwinger proper-time method modified for the case of the high temperature (see [10] for details). Thus, the average over the physical states values for the gluon PO and the Debye mass squared of charged gluons can be written as

$$\begin{aligned}
&\langle n,\sigma \mid \Pi^r_{ij} \mid n,\sigma\rangle = \Pi^r(P_4 = 0, h_r, T, n, \sigma) \\
&= \frac{g^2}{8\pi^{3/2}\beta}\, c_r \int_0^1 \frac{\mathrm{d}u}{\sqrt{u}} \int_0^\infty \frac{\mathrm{d}x}{\sqrt{x}}\, [g_r h_r \Delta]^{-1/2} \\
&\times \exp\Big\{-(2n+1)[\rho - x(1-u)] - 2y(1-u)\Big\} \\
&\qquad \times \Pi^r(x,u),
\end{aligned} \quad (22)$$

$$\begin{aligned}
&\Pi^r_{44}(P_4 = 0, h_r, T, n) \\
&= \frac{g^2}{8\pi^{3/2}\beta}\, c_r \int_0^1 \frac{\mathrm{d}u}{\sqrt{u}} \int_0^\infty \frac{\mathrm{d}x}{\sqrt{x}} [g_r h_r \Delta]^{-1/2} \\
&\times \exp\Big\{-(2n+1)[\rho - x(1-u)]\Big\} \widetilde{\Pi}^r(x,u),
\end{aligned} \quad (23)$$

where $x = g_r h_r u s$, $y = x\sigma$, $\sigma = \pm1$, $h_{r=1} = H_3$, $h_{r=2} = \lambda_+ H_8$, $h_{r=3} = \lambda_- H_8$, $g_{r=1} = g$, $g_{r=2,3} = \sqrt{\frac{3}{2}} g$,

$$\tanh\rho = \frac{(1-u)\sinh x}{(1-u)\cosh x + u\frac{\sinh x}{x}},$$

$$\Delta = (1-u)^2 + 2u(1-u)\frac{\sinh 2x}{2x} + u^2\frac{\sinh^2 x}{x^2}.$$

The explicit expressions of the functions $\Pi^r(x,u)$ and $\widetilde{\Pi}^r(x,u)$ are complicated and have in general the same form as in the case of $SU(2)$ gluodynamics considered in [10]. However, for our analysis we need in the asymptotic expansions of $\Pi^r(x,u)$ and $\widetilde{\Pi}^r(x,u)$ for the values of the parameters $u \sim 1$, and $x \gg 1$ corresponding to the high-temperature limit, $\frac{gH}{T^2} \ll 1$ [10]. Without loss of generality, the calculations can be carried out in the reference frame $P_3 = 0$. Performing the integrations we obtain

$$\begin{aligned}
&\Pi^{r=1}(P_4 = 0, P_3 = 0, H_3, T, n, \sigma = +1) \\
&= \frac{g^2}{4\pi}\sqrt{gH_3}\, T((4n + 11.44) + \mathrm{i}(10n + 7)), \\
&\Pi^{r=2,3}(P_4 = 0, P_3 = 0, H_3, H_8, T, n, \sigma = +1) \\
&= \frac{3g^2}{8\pi} c_{r=2,3} \left(\frac{3}{2}\right)^{\frac{3}{4}} \sqrt{\lambda_\pm}\,\sqrt{gH_8} \\
&\qquad \times T((4n + 11.44) + \mathrm{i}(10n + 7)), \\
&\Pi^{r=1}(P_4 = 0, P_3 = 0, H_3, T, n, \sigma = -1) \\
&= \frac{g^2}{4\pi}\sqrt{gH_3} \\
&\qquad \times T((4n + 15.62) + \mathrm{i}(2n + 9.69)), \\
&\Pi^{r=2,3}(P_4 = 0, P_3 = 0, H_3, H_8, T, n, \sigma = -1) \\
&= \frac{3g^2}{8\pi} c_{r=2,3} \left(\frac{3}{2}\right)^{\frac{3}{4}} \sqrt{\lambda_\pm}\,\sqrt{gH_8} \\
&\qquad \times T((4n + 15.62) + \mathrm{i}(2n + 9.69)), \\
&\Pi^{r=1}_{44}(P_4 = 0, P_3 = 0, H_3, T, n) \\
&= \frac{g^2T^2}{2} + \frac{g^2}{4\pi}\sqrt{gH_3}\, T((4n + 6) + \mathrm{i}(6n + 9)), \\
&\Pi^{r=2,3}_{44}(P_4 = 0, P_3 = 0, H_3, H_8, T, n) \\
&= g^2T^2 + \frac{3g^2}{8\pi} c_{r=2,3} \left(\frac{3}{2}\right)^{\frac{3}{4}} \sqrt{\lambda_\pm}\,\sqrt{gH_8} \\
&\qquad \times T((4n + 6) + \mathrm{i}(6n + 9)).
\end{aligned} \quad (24)$$

The sign "+" in the expressions (24) has to be taken for $r = 2$ and the sign "-" for $r = 3$ (r is the index of the charged basis (4)). From (19) and (24) it is seen that the real parts of the PO are positive in the ground and excited states. The imaginary parts in the expressions $\Pi(P_4 = 0, P_3 = 0, H, T, n, \sigma)$ and Π_{44} occur because of a non-analyticity of a number of terms in the integrands in the RHS of (22) and (23) for large $x \to \infty$. The integration contour ensuring the convergence of the x-integrations results in the imaginary parts in the expressions (24). The imaginary part describes the decay of the state owing to transitions to the states with lower energies. The first term in the expression Π_{44} is calculated by performing summation over the discrete frequencies k_4

and cannot be obtained from (23). The second one gives the next-to-leading term and is calculated by using the high-temperature static limit.

The imaginary part of Π_{44} for $n = 0$ describes the Landau damping of the ground state plasmon quasi-particles. We note that the imaginary parts entering the Π_{44} and the $\Pi(P_4 = 0, P_3 = 0, H, T, n = 0, \sigma = +1)$ are of the same order of magnitude. Since a spin interaction does not affect the former correction, and the tachyonic state in the field is excited just due to the spin interaction of the charged gluons, one has to conclude that the non-zero imaginary part of the latter function does not correspond to the instability of the chromomagnetic fields and also describes the usual damping of states at finite temperature. To verify whether or not the radiation corrections stabilize the spectrum at high temperature we calculate the gluon effective mass squared determined by the real part of the $\Pi(P_4 = 0, P_3 = 0, H, T, n = 0, \sigma = +1)$ at one-loop level. If this function is positive, the spectrum, and hence the vacuum, is stable.

5 Discussion

We have investigated the QCD restored phase. As we determined from the EP, accounting for the one-loop plus daisy diagrams, the vacuum with non-zero Abelian chromomagnetic fields H_3 and H_8 is favorable energetically. It is stable due to the magnetic masses of the charged gluons which have to be included into our consideration when the value of the background fields is estimated. These masses are computed from the gluon one-loop polarization operator in the external fields. As it occurred, the charged gluon spectrum in the fields accounting for the tree plus the one-loop corrections is stable in a wide interval of temperature above the T_{d}. It is important to notice that in the presence of a field the gluon magnetic mass $m^2_{\mathrm{magn.}} \sim g^2 (gH)^{1/2} T$ is generated in one-loop order, in contrast to what happens in the case of the trivial vacuum where the mass $m^2_{\mathrm{magn.}} \sim g^4 T^2$ is a non-perturbative effect [12, 15, 18]. Since the magnetic vacuum field is of the order $(gH)^{1/2} \sim g^2 T$, the charged gluon magnetic mass squared is estimated as $\sim g^4 T^2$. That is, the order of the magnetic mass is the same in both calculation methods. Clearly the former case is also non-perturbative because the field is taken into account exactly through the Green functions. If one accounts for the magnetic field perturbatively, a zero result follows [19]. Of course, the stabilization of the gluon spectrum by radiation corrections in the fields is an interesting fact which could not be expected beforehand. It was observed already in $SU(2)$ gluodynamics [20] and here for the $SU(3)$ gauge group. One may believe that this is the case for other non-Abelian gauge groups and therefore the stabilization is a reflection of the intrinsic field dynamics. Note here that another mechanism of field stabilization was discussed in [7], which takes into account the generation of the electrostatic gauge field potential (A_0 condensate). However this picture was not investigated consistently since the common generation of the A_0 and magnetic fields has not been considered. Some aspects of the influence of the A_0 condensate on the magnetic field have been investigated recently in one-loop order in [11].

Consider in more detail the values of the gluon magnetic mass determined in different calculations and compare these with the value of the vacuum magnetic field. In a recent paper [11], in $SU(2)$ gluodynamics to stabilize the one-loop effective potential the gluon magnetic mass $m_{\mathrm{magn.}}$ of order $\sim cg^2 T$ was introduced on heuristic grounds. Then, in particular, it was found that for $m_{\mathrm{magn.}} \geq 0.388 g^2 T$ the effective potential has a global minimum at $H = 0$. Hence it has been concluded that a sufficiently heavy magnetic mass leads to a trivial vacuum in the deconfinement phase. This critical value is close to the magnetic masses determined in a number of lattice simulations: $0.505\, g^2 T$ [21], $0.360\, g^2 T$ [22]. It is interesting to compare our result for the magnetic mass identified with the effective mass of gluon $M^2_{\mathrm{eff.}}(H) = 11,44 \frac{g^2}{4\pi} (gH)^{1/2} T - gH$ and the field $(gH)^{1/2} = \frac{g^2}{2\pi} T$ for the $SU(2)$ sector. A simple estimate gives $M_{\mathrm{eff.}} = 0.345 g^2 T$ which is close to the value derived by Philipsen. For this value we observed the stabilization of the magnetized vacuum. On the other hand, this value is insufficient to have a zero vacuum field, if the approach of the paper [11] is adopted. Moreover, if one takes into account the structure of the magnetic mass, $m^2_{\mathrm{magn.}} \sim \sqrt{gH} g^2 T$, there is no possibility to have zero for the generated field. So we believe that the magnetized vacuum has to be considered not as an artificial mathematical fact.

The gauge dependence of the gluon magnetic mass was discussed in Sect. 3. Now we would like to note some possible consequences of the gauge invariance of the ground state. As was mentioned in Sect. 3, this was investigated qualitatively by Feynman [17] for two-plus-one dimension gluodynamics and actually has a general character. Clearly, homogeneous magnetic fields break gauge invariance explicitly. Therefore they could not be considered as the true ground state. In fact, the solution derived in the present paper corresponds to some domain of the gauge invariant vacuum. The size of domains and their orientation could be determined by using the requirement of gauge invariance of the ground state at finite temperature. In [17] the zero temperature case is considered. To find the vacuum at finite temperature one also has to take into account the entropy of the states created. Most probably, a qualitative picture looks like a condensate of tubes with magnetic fluxes. Its detailed description needs a separate investigation.

Another important point which we are going to discuss is the influence of higher loop contributions. First we note that the one-loop plus daisy graphs account for the long distance contributions and give the main effect. This was realized already in the related problem on the electroweak phase transition in strong magnetic fields [23, 24]. The results obtained within this EP are in a good agreement with the one found in the non-perturbative approach in [25, 26] (see also the recent survey [27]). The most important feature of this approximation is that the EP is real at sufficiently high temperatures, and therefore the spontaneously generated magnetic fields are stable. We believe that the higher loop corrections will not change qualitatively the results obtained and the results on the stable magnetize vacuum survive. To check our results the resummations on the super daisy level have to be carried out. That is a problem for the future.

It is interesting to compare our results with those obtained in lattice calculations by Cea and Cosmai [28, 29]. In the former paper the creation of the color Abelian chromomagnetic field was investigated by means of the lattice Schrödinger functional. It was observed that at $T = 0$ the applied external chromomagnetic field is completely screened by the vacuum. At finite temperature the applied field is supported by the temperature and increased with the growth of temperature. That is in correspondence with our calculations. In the latter paper the influence of the external fields on the deconfinement phase transition has been investigated and an intimate connection between Abelian chromomagnetic field and color confinement was observed. This interesting result is not directly related with the one obtained in the present paper, because we do not consider the field as an external one. From our results it follows that in the deconfinement phase the Abelian chromomagnetic fields have to be present. So, we have to answer the question how the spontaneous vacuum magnetization affects the temperature of the phase transition.

References

1. M.N. Chernodub, F.V. Gubarev, M.I. Polikarpov, V.I. Zakharov, Phys. Atom. Nucl. **64**, 561 (2001)
2. J. Stack, S. Neiman, R. Wensley, Phys. Rev. D **50**, 3399 (1994)
3. H. Shiba, T. Suzuki, Phys. Lett. B **333**, 461 (1994)
4. G. Bali, V. Bornyakov, M. Müller-Preussker, K. Schilling, Phys. Rev. D **54**, 2863 (1994)
5. Y.M. Cho, D.G. Park, Phys. Rev. D **65**, 074027 (2002)
6. K. Enqvist, P. Olesen, Phys. Lett. B **329**, 195 (1994)
7. A.O. Starinets, A.S. Vshivtsev, V.Ch. Zhukovsky, Phys. Lett. B **322**, 403 (1994)
8. V.V. Skalozub, Int. J. Mod. Phys. A **11**, 5643 (1996)
9. V. Skalozub, M. Bordag, Nucl. Phys. B **576**, 430 (2000)
10. V.V. Skalozub, A.V. Strelchenko, Phys. Atom. Nucl. **63**, 1956 (2000)
11. P. Meisinger, M. Ogilvie, Phys. Rev. D **66**, 105006 (2002)
12. O.K. Kalashnikov, Fortschritte Phys. **32**, 325 (1984)
13. F.J. Yndurain, Quantum chromodynamics (Springer-Verlag, Berlin 1983)
14. J. Schwinger, Phys. Rev. D **7**, 1696 (1973)
15. D. Buchmüller, O. Philipsen, Phys. Lett. B **354**, 403 (1995)
16. R. Kobes, G. Kunstatter, A. Rebhan, Phys. Rev. Lett. **64**, 2992 (1990)
17. R.P. Feynman, Nucl. Phys. B **188**, 479 (1981)
18. U.M. Heller, F. Karsch, J. Rank, Phys. Lett. B **355**, 511 (1995)
19. P. Elmfors, D. Persson, Nucl. Phys. B **538**, 309 (1999)
20. V.V. Skalozub, A.V. Strelchenko, in The Casimir Effect Fifty Years Later, Proceedings of IV International Conference on Quantum Field Theory under the Influence of External Condition, edited by M. Bordag (World Scientific 1999) p. 229
21. A. Cucchieri, F. Karsch, P. Petreczky, Phys. Lett. B **497**, 80 (2001)
22. O. Philipsen, Nucl. Phys. B **628**, 167 (2002)
23. V.V. Skalozub, M. Bordag, Int. J. Mod. Phys. A **15**, 349 (2000)
24. V.V. Skalozub, V.I. Demchik, Ukranian Phys. J. **46**, 784 (2001)
25. K. Kajantie, M. Laine, K. Rummukainen, M. Shaposhnikov, Nucl. Phys. B **458**, 90 (1996)
26. K. Kajantie, M. Laine, J. Peisa, K. Rummukainen, M. Shaposhnikov, Nucl. Phys. B **544**, 357 (1999)
27. D. Grasso, H.R. Rubinstein, Phys. Rept. **348**, 163 (2001)
28. P. Cea, L. Cosmai, Abelian chromomagnetic background field at finite temperature on the lattice, hep-lat/0101017
29. P. Cea, L. Cosmai, Abelian chromomagnetic fields and confinement, hep-lat/0204023